- Please check that this question paper contains **11** printed pages.

- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

- Please check that this question paper contains **29** questions.

- **Please write down the Serial Number of the question before attempting it.**

- 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

MATHEMATICS

Time allowed : 3 hours *Maximum Marks : 100*

General Instructions :

(i) ***All*** *questions are compulsory.*

(ii) *The question paper consists of* ***29*** *questions divided into three sections A,*

B and C. Section A comprises of ***10*** *questions of* ***one mark*** *each, Section B*

comprises of ***12*** *questions of* ***four marks*** *each and Section C comprises of* ***7*** *questions of* ***six marks*** *each.*

(iii) *All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.*

(iv) *There is no overall choice. However, internal choice has been provided in*

4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

(v) *Use of calculators is* ***not*** *permitted. You may ask for logarithmic tables, if required.*

SECTION A

Question numbers 1 to 10 carry 1 mark each.

1. If R = {(x, y) : x + 2y = 8} is a relation on N, write the range of R.
2. If $\tan^{-1} x + \tan^{-1} y = \pi/4$ xy < 1, then write the value of x + y + xy.
3. If A is a square matrix such that $A^2 = A$, then write the value of 7A – $(I + A)^3$ where I is an identity matrix.
4. If $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$ find the value of x + y.
5. If $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$, find the value of x.
6. If $f(x)=\int_0^x t \sin t \, dt$, then write the value of f□(x).
7. Evaluate : $\int_2^4 \frac{x}{x^2+1} dx$
8. Find the vaule of 'p' for which the vectors $3\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$ and $\hat{\imath}$-2p $\hat{\jmath} + 3\hat{k}$ are parallel.
9. Find $\vec{a}.(\vec{b} * \vec{c}), if \ \vec{a} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}, \vec{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k} \ and \ \vec{c} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}.$
10. If the cartesian equations of a line are $\frac{3-x}{5} = \frac{y+4}{1} = \frac{2z-6}{4}$, write the vector equation for the line.

SECTION B

11. If the function f : R □ R be given by $f(x) = x^2 + 2$ and g : R □ R be given by $g(x) = \frac{x}{x-1}, x \neq 1$ find fog and gof and hence find fog (2) and gof (– 3).
12. Prove that $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right] = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x, \frac{-1}{\sqrt{2}} \leq x \leq 1$

OR

If $\tan^{-1}\left(\frac{x-2}{x-4}\right)+\tan^{-1}\left(\frac{x+2}{x+4}\right)=\frac{\pi}{4}$, find the value of x.

13. Using properties of determinants, prove that

$$\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix}=x^3$$

14. Find the value of $\frac{dy}{dx}$ $at\ \theta=\frac{\pi}{4}, if\ x=ae^{\theta}(\sin\theta-\cos\theta)\ and\ y=ae^{\theta}(\sin\theta+\cos\theta)$

15. If y= $Pe^{ax}+Qe^{bx}, show\ that$

$$\frac{d^2y}{dx^2}-(a+b)\frac{dy}{dx}+aby=0$$

16. Find the value(s) of x for which y =[x (x – 2)]^2 is an increasing function.

OR

Find the equations of the tangent and normal to the curve at $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

The point ($\sqrt{2a}$,b).

17. Evaluate :

$\int_0^{\pi}\frac{4x\sin x}{1+cos^2 x}$dx

or

Evaluate : $\int\frac{x+2}{\sqrt{x^2+5x+6}}dx$

18. Find the particular solution of the differential equation $\frac{dy}{dx}=1+x+y+xy$, given that y=0 when x=1.

19. Solve the differential equation $(1+x)^2\frac{dy}{dx}$+y=$e^{\tan^{-1}x}$

20. Show that the four points A,B,C and D with position vectors $4\hat{\imath}+5\hat{\jmath}+\hat{k}, \hat{\jmath}-\hat{k}, 3\hat{\imath}+9\hat{\jmath}+\hat{k}\ and\ 4\left(-\hat{\imath}+\hat{\jmath}+\hat{k}\right)$ respectively are coplanar.

OR

The scalar product of vector $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ with a unit vector along the sum of vectors $\vec{b} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ and $\vec{c} = \lambda\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ $is\ equal\ to\ one. find\ the\ value\ of$ λ and hence find the unit vector along $\vec{b} + \vec{c}$.

21. A line passes through (2,-1,3) and is perpendicular to the lines $\vec{r} = \left(\hat{\imath} + \hat{\jmath} - \hat{k}\right) + \lambda\left(2\hat{\imath} - 2\hat{\jmath} + \hat{k}\right) and\ \vec{r} = \left(2\hat{\imath} - \hat{\jmath} - 3\hat{k}\right) + \mu(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$. Obtain its equation in vector and Cartesian form.
22. An experiment succeeds thrice as often as it fails. Find the probability that in the next five trials, there will be at least 3 successes.

SECTION C

23. Two schools A and B want to award their selected students on the values of sincerity, truthfulness and helpfulness. The school A wants to award < x each, < y each and < z each for the three respective values to 3, 2 and 1 students respectively with a total award money of < 1,600. School B wants to spend < 2,300 to award its 4, 1 and 3 students on the respective values(by giving the same award money to the three values as before). If the total amount of award for one prize on each value is < 900, using matrices, find the award money for each value. Apart from these three values, suggest one more value which should be considered for award.
24. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$.also show that the maximum volume of the cone is $\frac{8}{27}$ of the volume of the sphere.
25. Evaluate : $\int \frac{1}{cox^4 x + sin^4 x} dx$
26. Using integration, find the area of the region bounded by the triangle whose vertices are (–1, 2), (1, 5) and (3, 4).
27. Find the equation of the plane through the line of intersection of the planes x+ y + z = 1 and 2x + 3y + 4z = 5 which is perpendicular to the plane x – y + z= 0. Also find the distance of the plane obtained above, from the origin.

OR

Find the distance of the point (2, 12, 5) from the point of intersection of the line $\vec{r} = 2\hat{\imath} - 4\hat{\jmath} + 2\hat{k} + \lambda(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k})$ and the plane $\vec{r}.(\hat{\imath} - 2\hat{\jmath} + \hat{k})$=0.

28. A manufacturing company makes two types of teaching aids A and B of Mathematics for class XII. Each type of A requires 9 labour hours of fabricating and 1 labour hour for finishing. Each type of B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available per week are 180 and 30 respectively. The company makes a profit of < 80 on each piece of type A and < 120 on each piece of type B. How many pieces of type A and type B should be manufactured per week to get a maximum profit ? Make it as an LPP and solve graphically. What is the maximum profit per week?

29. There are three coins. One is a two-headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the times and third is also a biased coin that comes up tails 40% of the times. One of the three coins is chosen at random and tossed, and it shows heads. What is the probability that it was the two-headed coin ?

OR

Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find the probability distribution of the random variable X, and hence find the mean of the distribution.